ON THE CALCULATION OF THE DISTRIBUTION OF HEAT LIBERATION RATES IN NUCLEAR REACTOR COMPONENTS

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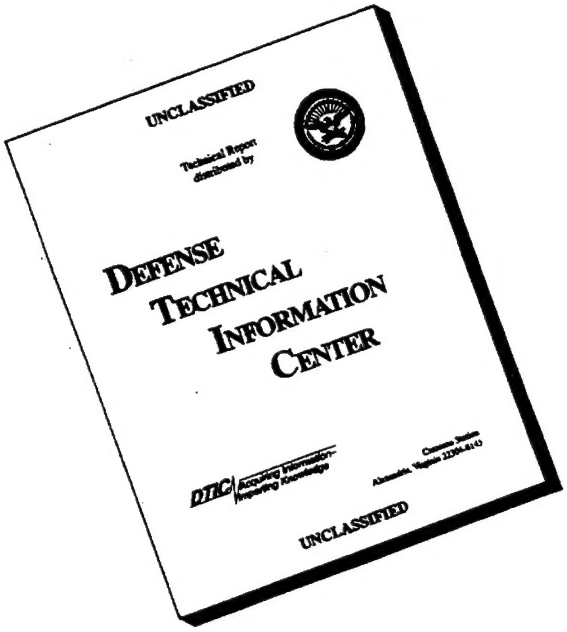

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ABSTRACT

Knowledge of local heating rates is needed for estimation of operating temperatures, thermal stresses, and cooling requirements in nuclear reactor components. Heat is liberated by the dissipation of the energy of fission fragments, beta particles, fast neutrons, and gamma photons. Heating rates are formulated in terms of either neutron or gamma flux densities, the corresponding collision probabilities, and appropriate energy transfer functions, the forms of which are derived. Special methods of estimating the flux densities are discussed. The data on the magnitudes of the various energy sources are reviewed.

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By
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INTRODUCTION

The designer of nuclear reactors needs to know the local rates of heat liberation in the various reactor components in order to estimate temperatures, cooling requirements, thermal stresses, corrosion rates, radiation damage annealing rates, temperature dependant reactivity changes, and so on. The heat liberation rates can be formulated in terms of the local neutron and gamma fluxes, ϕ_n and ϕ_γ. Admittedly, values of these are not easily come by. Nevertheless, recent progress in neutron and gamma particle transport theories makes it possible to treat many systems of engineering interest. It is the purpose of this paper to formulate the heat liberation rates and to indicate methods of estimating the fluxes.

ENERGY SOURCES AND MODES OF LOCAL DEPOSITION OF HEAT

Heat is liberated in reactor components by the dissipation of the kinetic energy of fission fragments, beta particles, gamma photons, and neutrons. These particles, in penetrating matter, collide with electrons and atomic nuclei and impart energy to them. In some collosion processes, part of this energy is reradiated, but for the most part it is rather rapidly degraded into thermal motion of nuclei and electrons in the macroscopically-near vicinity of the collision.

Fission fragments, primary gamma photons, and neutrons result directly from fission. Fission fragments emit beta particles and gamma photons during radioactive decay. Neutrons diffuse through the reactor and generate gamma photons by inelastic collisions with, and absorptions in various nuclei. Such activated nuclei often decay, emitting beta and gamma particles. Photons arising from all these processes may experience many scattering collisions, losing part of their energy at each collision.

FISSION FRAGMENT AND BETA DECAY ENERGY

According to Shapiro (16), two and occasionally three heavy particles result from the fission of U^{235} by thermal neutrons. Hanna (9) critically examined the data as of 1950 and concluded that the most probable value for the average kinetic energy of the fission fragments, G_{ff}, lies in the range 168 ± 5 mev per fission.

Because of their high charge, fission fragments have short ranges. (16). Accordingly, their energy, G_{ff}, is dissipated, macroscopically at the point of fission.

K. Way (21) estimated the energy released by the beta decay of fission fragments, $G_{ff\beta}$, to be 7 mev per fission. Effective ranges are of the order of 1 mm. or less in reactor materials; hence, the beta energy is also liberated very near the point of fission.

The sum of the fission fragment and beta ray energies is thus about 175 mev per fission. The heat release rate is proportional to the number of fissions per unit volume per unit time and may be formulated as follows:

$$(G_{ff} + G_{ff\beta}) \frac{\text{watts}}{\text{cc.}} = \int_{E_n = 0}^{E_n = \infty} \frac{m \text{ gm. } U^{235}}{\text{cc.}} \times \frac{\text{mole } U^{235}}{235 \text{ gm. } U^{235}} \times \frac{6.02 \times 10^{23} \text{ atoms}}{\text{mole}}$$

$$\times \sigma_f (E_n) \frac{\text{barns}}{\text{atom } U^{235}} \times \frac{10^{-24} \text{ Fissions}}{\text{barn } \left(\frac{\text{neutron-cm}}{\text{cm}^3}\right)} \times$$

$$\phi_n(E_n) \frac{\text{neutron-cm.}}{\text{cm.}^3 \text{ sec. mev.}} \times \frac{175 \text{ mev.}}{\text{fission}} \times \frac{1.65 \times 10^{-13} \text{ watt-sec}}{\text{mev.}}$$

(1.609)

$$\times d E \text{ mev.}$$

Here, $\phi_n (E_n)$ represents the energy dependent neutron flux-- i.e., total path length (cm) traversed by neutrons having energy E_n (mev) per unit volume (cm.^{3}) per unit time (sec.) per unit energy range (mev), The integration is carried over all energies; however, there are negligibly few neutrons having energies less than that corresponding to the temperature of the medium (30° C. ~ 0.025 ev.), or greater than 10 mev. In "thermal" reactors, the great majority of the flux is concentrated into a narrow band around the temperature of the medium. Furthermore, the fission cross section, $\sigma_f(E_n)$, decreases rapidly with increasing neutron energy. Accordingly, in thermal reactors, the integral is approximated very well by the product of the "thermal" flux (say the flux lying between 0.025 and 0.1 mev) and the fission cross section evaluated at the temperature of the medium. Let the thermal flux be denoted by ϕ_n (th), whence

$$\phi_n (th) = \int_{0.025}^{0.1} \phi_n (E_n) \, dE_n \qquad 2$$

The product of the first five quatities of Equation 1 is commonly denoted by $\Sigma_f(E_n)$, called the "macroscopic fission cross section". It is the probability, per cm. of travel, that a neutron of energy E will experience

a fission collision, and has units of fissions per neutron-cm. Using Σ_f, Equation 2, and converting to BTU/hr.ft.^{3}, Equation 1 takes the form, for thermal reactors

$$G_f = (G_{ff} + G_{ff\beta}) \frac{BTU}{hr.ft^3} = 2.70 \times 10^{-6} \Sigma_f(th)\, \phi_n\,(th) \qquad 3$$

For U^{235}, σ_f equals 549. Taking m to be 1 gm. U^{235}/ cc. gives α Σ_f of 1.40 fissions/neutron-cm. At a thermal flux level, ϕ_n(th), of 10^{13} neutron-cm./cm.^{3} sec., the heat production is about 400 watts/cc or 40 x 10^6 BTU/hr.ft.^{3}

NEUTRON ENERGY

Neutrons impart kinetic energy to atomic nuclei by three processes; (a) absorption, (b) elastic scattering, (c) inelastic scattering.

In an absorption collision, the neutron enters and combines with the target nucleus, forming the next higher isotope. The new nucleus is "knocked on", and its velocity is readily calculated from a simple momentum balance.

The thermal motion of the nucleus prior to the collision may be neglected, since it is very small compared to the neutron velocities of interest. Therefore,

$$mV_n = (M+m)\, V_{N'} \qquad 4$$

where m = atomic mass of neutron ($m \approx 1$)

M = atomic mass of nucleus

V_n = velocity of neutron

$V_{N'}$ = velocity of nucleus after collision

By definition, the kinetic energies of the nucleus and neutron are, respectively

$$E_{N'} = \tfrac{1}{2}(M+m)\, V_{N'}^2 \qquad 5a$$

$$E_n = \tfrac{1}{2} m V_n^2 \qquad 5b$$

Combining Equations 4, and 5

$$E_{N'} = mE_n/(M+m) \qquad 6$$

Defining δ_{nc} as the fraction of the kinetic energy of the incident neutron appearing as kinetic energy of the target nucleus after a capture collision, one has

$$\delta_{nc} = E_{N'}/E_n = m/(M+m) \qquad 7$$

The remainder of the neutron kinetic energy plus the binding energy is released in the form of a series of so-called "capture" gamma photons of decreasing energy. The N' isotope may be radioactive, and may decay by alpha, beta, and gamma emission. Later sections deal with heat release from these sources.

The local heating effect due to the capture of fast neutrons is thus

$$G_{nc} = \int_{E_o}^{\infty} \delta_{nc}\, E_n\, \Sigma_{nc}(E_n)\, \phi_n(E_n)\, dE_n \qquad 8$$

where Σ_{nc} is the so-called macroscopic neutron-capture cross section. It is the probability, per unit length of travel, that a neutron will experience a capture collision. The lower limit of integration, E_o, is chosen suffiently high that Equation 8, derived on the assumption that the energy of the target nucleus prior to the collision is negligible in comparison with the initial neutron energy E_n, remains valid, but also sufficiently low that only a negligible portion of the neutron energy remains to be transferred at energies less than E_o. The choice depends somewhat on the spectral distribution of the fast neutron flux, but values between 10 and 100 ev are nearly always satisfactory.

If two or more kinds of atoms are present in the scattering medium, the corresponding G_{nc} must be summed over all of these. This is possible because the capture cross sections, Σ_{nc}, are additive.

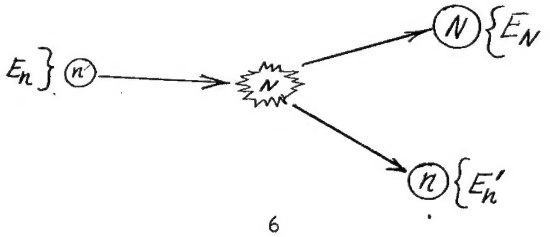
Both momentum and kinetic energy are conserved in elastic collisions. The appropriate balances are most conveniently formulated in the center-of-mass system of coordinates. By combining these and transforming the results to the laboratory system of coordinates, Glasstone and Edlund (5) obtain a result that may be put into the form

$$E_n'/E_n = \frac{M^2 + 2Mm\cos\theta + m^2}{(M + m)^2} \qquad 9$$

where θ is the angle of scattering in the center-of-mass system. Since kinetic energy is conserved in the collision, it follows that

$$E_N/E_n = 1 - E_n'/E_n \qquad 10$$

The scattering is isotropic in the center-of-mass system, hence the



probability $P(\theta)d\theta$ that a neutron will be scattered elastically through an angle θ into angle $d\theta$ is related (5) to the angle θ by

$$P(\theta)d\theta = \text{Sin}\,\theta d\theta/2 \qquad 11$$

By definition, the average fraction δ_{ne} of neutron energy E_n converted into target nucleus kinetic energy E_n' is given by the integral

$$\delta_{ne} = \int_0^{\pi} (1 - E_n'/E_n)\, P(\theta)\, d\theta \qquad 12$$

which leads to the simple result that

$$\delta_{ne} = (1 - \alpha)/2 \qquad 13$$

where $$\alpha = \left[(M - m)/(M + m)\right]^2 \qquad 14$$

The elastic collision heating rate is formulated similarly to Equation 13.

$$G_{ne} = \int_{E_o}^{\infty} \delta_{ne}\, E_n\, \Sigma_{ne}(E_n)\, \phi_n(E_n)\, dE_n \qquad 15$$

The lower limit E_o again lies in the range 10 to 100 ev. As with G_{nc}, G_{ne} must be summed with respect to all kinds of atoms present.

If the compound nucleus formed by the interaction of a neutron with a target nucleus decomposes by the emission of another neutron, the net result is equivalent to the scattering of the incident neutron. The nucleus, however, may be left in an excited state, in which case kinetic energy is not conserved and the scattering is inelastic. There is then, for this case, no relation between the scattering angle and the energies of the incident and scattered neutrons. Instead, with heavy ($M > 40$), non-magic nuclei, the probability $P(E_n, E_n', Z)dE_n'$ that a neutron of energy E_n will be scattered through an energy range $(E_n - E_n')$ into energy range dE'_n by a nucleus of atomic number Z, is given by the Weiskopf (7) formula, which may be put into the form

$$P(E_n,E_n',Z)dE_n' = \frac{(E_n'/T_Z)\exp(-E_n'/T_Z)}{1-(1+E_n/T_Z)\exp(-E_n/T_Z)} \qquad 16$$

where T_Z is the so-called "nuclear temperature" which must be determined experimentally for each nucleus. Values are listed by Bjorklund (2) who reports that T_Z is independent of E_n and E_n' and is a function of only Z, the atomic number.

The scattering is isotropic in the center-of-mass system of coordinates. However, with heavy nuclei, it is also approximately isotropic in the laboratory system. Accordingly, the probability $P(\Theta)\,d\Theta$ that the neutron will be scatted through angle Θ into angle $d\Theta$ is given by Equation 11, where Θ is now the scattering angle in the laboratory system. Specifying E_n, E_n', and Θ completely determines E_N, the kinetic energy of the target nucleus (for the case of negligible initial energy). A simple momentum balance readily yields the relation

$$E_N/E_n = (1/M)\left[1 - 2\,(E_n'/E_n)^{1/2}\cos\Theta + (E_n'/E_n)\right] \qquad 17$$

By definition, the fraction δ_{ni} of incident neutron energy E_n that appears as kinetic energy E_N of the target nucleus is given by

$$\delta_{ni}(E_n,Z) = \int_0^{E_n}\int_0^{\pi}(E_N/E_n)P(E_n,E_n')\Sigma_{ni}(E_n,Z)\,dE_n'\,P(\Theta)d\Theta \qquad 18$$

where $\Sigma_{ni}(E_n,Z)$ is the inelastic scattering "cross section".

Substitution from Equations 11, 16, and 17 followed by integration gives

$$\delta_{ni}(E_n,Z) = \frac{\Sigma_{ni}(E_n,Z)T_Z\left[(1+2T_Z/E_n) - (3+2T_Z/E_n+2E_n/T_Z)\exp(-E_n/T_Z)\right]}{M\left[1-(1+E_n/T_Z)\exp(-E_n/T_Z)\right]} \qquad 19$$

Equation 19 is valid only for heavy, non-magic nuclei. For light and magic nuclei, the possible energy levels in the excited target nucleus are more or less discrete, and accordingly the probability $P(E_n,E_n')$ is distributed in discrete bands which must be determined experimentally for each nucleus (7). The data are then applied to Equation 19 in place of Equation 16.

For all nuclei, $\delta_{ni}(E_n,Z)$ is a function of E_n. The local heating rate due to inelastic scattering of fast neutrons accordingly takes the form

$$G_{ni} = \int_{E_o}^{\infty} \delta_{ni}(E_n,Z) E_n \Sigma_{ni}(E_n) \phi_n(E_n)\, dE_n \qquad 20$$

where E_o is the threshold energy for inelastic scattering. G_{ni} must be summed with respect to all kinds of atomic nuclei present.

ACTIVATED NUCLEI BETA DECAY ENERGY

Nuclei rendered unstable by the absorption of a neutron may emit alpha, beta, or other particles as well as gamma photons. Beta emission is the most common mode of decay. For a given transition, the beta particles are emitted with energies ranging continuously from zero up to some characteristic maximum, E_e (max), which is the most commonly tabulated value (10). Since the contribution to the local heating from beta decay of activated nuclei is usually small compared to other contributions, it is generally sufficiently precise (15) to take the average electron energy to be one third E_e (max). Thus, the local heating rate due to "neutron-capture" betas becomes

$$G_{nc\beta} = \int_{o}^{\infty} \Sigma_{nc}(E_n) \phi_n(E_n)\, dE_n \sum_i n_i E_{ei}(max)/3 \qquad 21$$

?? where n_i is the number of betas of characteristic energy E_{ei} (max) emitted in the ith portion of the decay chain per neutron capture. Since isomeric transitions occur, the n_i are numbers equal to 1 or less. The decay process may proceed through several atomic numbers. Note that the contribution from "thermal" neutrons in Equation 21 may be large and override the contribution from fast neutrons.

GAMMA PHOTON ENERGY

Gamma photons penetrating matter exhibit three interaction processes: (a) photoelectric effect, (b) pair production, and (c) Compton scattering.

In the first process, a photon transfers all of its energy to an electron, which is "knocked on" and dissipates its energy in the immediate vicinity. The probability, per unit length of travel, that a photon will experience such a collision is denoted by $\mu_{pe}(E_\gamma)$. When the atom from which the electron was ejected acquires another electron in its place, an x-ray of low energy is emitted; however, about 95 per cent of these are reabsorbed inside the atom, and an "Auger" electron of low energy is ejected from among the outer valence electrons. This electron also dissipates its energy in the vicinity. Various other energy "dribbling" processes may occur, but the net effect is that all of the energy of the primary gamma photon is dissipated as heat in the vicnity of the collision. Thus the fraction $\delta_{\gamma pe}$ of initial gamma energy E_γ dissipated is unity. Consequently, the heating rate due to photoelectric collisions of gamma photons is given by

$$G_{\gamma pe} = \int_0^\infty E_\gamma \mu_{pe}(E_\gamma)\phi_\gamma(E_\gamma)dE_\gamma \qquad 22$$

where E_γ denotes the energy of the gamma photon, and $\phi_\gamma(E_\gamma)$ denotes the energy dependent gamma flux.

Photons having energies in excess of 1.02 mev may interact with the coulomb field of a nucleus and produce an "electron pair".. The probability, per unit length of travel, of pair production is denoted by $\mu_{pp}(E_\gamma)$. The resulting beta particles dissipate their kinetic energy near the point of production. The positron ultimately combines with an electron forming two photons having energies approximately 0.51 mev. These may penetrate some distance from the point where the pair production occurred. The nucleus involved in the collision acquires some energy also. The net result is that $(E_\gamma - 1.02)$ mev of energy is dissipated at the point of collision. Thus $\delta_{\gamma pp}$, the fraction of photon energy E_γ dissipated locally, becomes

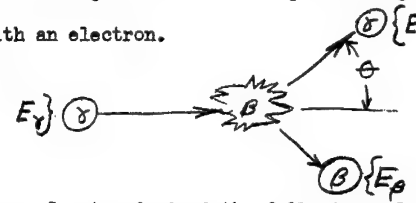
$$\delta_{\gamma pp} = (E_\gamma - 1.02)/E_\gamma \quad ; E_\gamma \geq 1.02 \qquad 23$$

The heating rate due to pair production takes the form

$$G_{\gamma pp} = \int_0^\infty \delta_{\gamma pp} E_\gamma \mu_{pp}(E_\gamma) \phi_\gamma (E_\gamma) dE_\gamma \qquad 24$$

The lower limit is taken as zero for later convenience, but $\mu_{pp}(E_\gamma)$ is zero for E_γ less than 1.02.

Compton scattering is the scattering resulting from an elastic collision of a photon with an electron.



For this process, Compton derived the following relation between the energies E_γ and E'_γ of the photon before and after collision and the

scattering angle Θ.

$$E'_\gamma / E_\gamma = \frac{1}{1 + E_\gamma (1 - \cos\Theta)/0.51} \qquad 25$$

Since the collision is elastic, kinetic energy is conserved, and

$$\frac{E_\beta}{E_\gamma} = \frac{E_\gamma - E'_\gamma}{E_\gamma} = 1 - \frac{E'_\gamma}{E_\gamma} \qquad 26$$

The probability that a photon will be scattered through an angle Θ into angle $d\Theta$ is given by the Klein-Nishina formula (8), which may be put into the form

$$P(\Theta)d\Theta = K(E'_\gamma / E_\gamma)2 \left[(E_\gamma / E'_\gamma) + (E'_\gamma / E_\gamma) - \sin^2\Theta \right] \sin\Theta \, d\Theta \qquad 27$$

where K is a normalization constant such that

$$\int_0^\pi P(\Theta)d\Theta = 1 \qquad 28$$

and may be evaulated by substitutions from Equations 25 and 27 followed by integration.

The fraction $\delta_{rc}(E_\gamma)$ of energy E_γ of the incident photon that appears as kinetic energy E_β of the beta particle is given by

$$\delta_{rc}(E_\gamma) = \int_0^\pi (1 - E'_\gamma / E_\gamma) E_\gamma P(\Theta)d\Theta \qquad 29$$

Substitution from Equations 25, 26 and 27 yields an integral that has not been put into a convenient algebraic form but has been evaluated (See Equation 32 et. seq.).

The energy deposition due to Compton scattering may be formulated

as follows

$$G_{\gamma c} = \int_0^\infty \delta_{\gamma c}(E_\gamma)/\mu_c\,(E_\gamma)\, E_\gamma \phi_\gamma(E_\gamma)\, dE_\gamma \qquad 30$$

Summing the gamma heating contributions given by Equations 29, 24 and 22,

$$G_\gamma = \int_0^\infty (\delta_{\gamma pe}/\mu_{pe} + \delta_{\gamma pp}/\mu_{pp} + \delta_{\gamma c}/\mu_c)\, E_\gamma \phi_\gamma(E_\gamma)\, dE_\gamma \qquad 31$$

where the E_γ 's have been omitted in the parenthesis.

Substituting from Equation 23 and rearranging, but, keeping in mind that $\mu_{\gamma pp}$ vanishes for energies less than 1.02 mov. and that $\delta_{\gamma pe}$ is unity, gives

$$G_\gamma = \int_0^\infty (\mu_{pe} + \mu_{pp} + \delta_{\gamma c}/\mu_c)\, E_\gamma \phi_\gamma(E_\gamma) dE_\gamma - \int_{1.02}^\infty 1.02 \mu_{pp} \phi_\gamma(E_\gamma) dE \qquad 32$$

The quantity $(\mu_{pe} + \mu_{pp} + \delta_{\gamma c}/\mu_c)$* appearing in the left hand integral, divided by the density of the medium, has been graphed by Snyder and Powell (18) as a function of E_γ for various materials. Their coefficient $\mu - \sigma_s$ is identical with the quantity in the parenthesis. It is called the energy absorption coefficient. If the so-called "annihilation photons", resulting from the neutralization of the positron, are assumed to be absorbed in the vicinity of the point of pair production, one may neglect the right-hand integral provided the annihilation photons are not included in the gamma flux $\phi_\gamma(E_\gamma)$.

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* Denoted elsewhere by $\mu_e(E_\gamma)$.

NEUTRON FLUX DISTRIBUTION

The neutron diffusion and transport theories yield descriptions of the spectral and spatial distributions of the neutron fluxes, $\phi_n(E_n)$. These theories have been discussed exhaustively elsewhere (5) and will not be considered here in detail. A one or two-group diffusion calculation will suffice to describe the "gross" variation of the thermal flux in many thermal reactors with sufficient precision for heat generation calculations. Let $\phi_n(th,\underline{0})$ be the flux at some reference point in a homogeneous reactor (say the center), and let $f(\underline{r})$ denote a function, obtained from diffusion theory, describing the spatial variation of $\phi_n(th,\underline{r})$, where $\underline{r}$ denotes the radius vector from the reference point, such that

$$\phi_n(th,\underline{r}) = \phi_n(th,\underline{0})\ f(\underline{r}) \qquad 33$$

By Equation 3, it follows (using the notation of Equation 33) that

$$G_f(\underline{r}) = A\,\phi_n(th,\underline{0})\ f(\underline{r})\ \Sigma_f(th,\underline{r}) \qquad 34$$

where A is a proportionality factor. The total heat output from the reactor due to fission fragments, Q_f, is obtained by integrating $G_f(\underline{r})$ over the volume of the fuel-bearing part of the core.

$$Q_f = \int_{V_f} G_f(\underline{r})\, dV \qquad 35$$

where V_f denotes the volume of the core.

Substituting from Equation 34 and solving for $A\,\phi_n(th,\underline{0})$ gives

$$A\,\phi_n(th,\underline{0}) = Q_f \Big/ \int_{V_f} f(\underline{r})\Sigma_f(th,\underline{r})\, dV \qquad 36$$

Putting this back into Equation 34 gives

$$G_f(\underline{r}) = Q_f \; f(\underline{r}) \Sigma_f(th,\underline{r}) \Big/ \int_{V_f} f(\underline{r}) \Sigma_f(th,\underline{r})\, dV \qquad 37$$

The energy available from fission fragments (kinetic and beta decay) is about 175 mev. per fission. The total energy per fission is about 200 mev. Thus Q_f is about 87 per cent of Q_t, the total reactor heat output. The balance of Q_t is generated by fission gammas (8 mev), fission fragment decay gammas (6 mev), fast neutrons (5 mev), capture gammas and decay betas (7 mev). The particles bearing this energy (about 25 mev) tend to leak out of the fuel-bearing core. A goodly portion of their energy is released in the coolant, reflector, thermal shield, pressure shell, and biological shield. However, in designing the fuel-bearing core, it is sufficient and conservative to assume that all of the reactor heat, Q_t, of a homogeneous reactor is released in the fuel, and that it has the same spatial distribution as Q_f.

The function $\underline{f}(\underline{r})$ has been evaluated by Glasstone and Edlund (5) for certain bare, homogeneous, thermal reactors of simple shape, with $\Sigma_f(th,\underline{V})$ uniform.

Table 1

Geometry	$f(\underline{r})$	Go/Gav
Sphere of radius R	$\sin(\pi r/R)/(\pi r/R)$	3.29
Cube of side A	$\cos(\pi x/A)\cos(\pi y/A)\cos(\pi z/A)$	3.64
Right cylinder of radius R	$J_o(2.405r/R)\cos(\pi z/2R)$	3.87

By integrating the given functions over the corresponding geometric forms, the ratio of the maximum heat release rate, G_o, to the average rate, G_{av}, is readily obtained. Values are listed in Table 1. The presence of moderating reflectors, control elements, structural materials, coolants, ducts, and non-uniform loading (variable Σ_f) modify these simple distributions. Space does not permit treatment of these effects, but it is important to reduce G_o/G_{av} so that a larger fraction of the core may operate at or near the limiting conditions and burn-up of nuclear fuel and accompanying effects will be more uniform.

Glasstone and Edlund point out that the functions $J_o(x)$ and $(\text{Sin } x)/x$ are rather similar to Cos x, and to a good first approximation may be replaced by Cos x.

In order to increase resonance escape in low enrichment uranium reactors, the fuel is sometimes segregated from the moderator. Fast neutrons, released by fission in the fuel, have a high probability of escaping into the moderator where they are quickly slowed down without exposure to resonance absorption in U^{238}. In the ORNL Graphite Reactor, this result is achieved by using metallic fuel rods about 1 inch in diameter arranged in an 8 inch square array in a matrix of graphite. The fuel has negligible moderating properties; neutrons are "thermalized" almost entirely in the graphite and diffuse thence back into the fuel, there causing further fissions. As a result, there is a local gradient in the flux density both in the moderator and in the fuel superimposed on the gross flux of Eq. 33. Some reactors, which are heterogeneous from a phase standpoint, such as the MTR, are really homogeneous from a nuclear standpoint, there being negligible gradients in the neutron flux in the moderator and fuel.

Neutron diffusion theory may be employed in the calculation of thermal flux distributions in fuel elements in heterogeneous reactors. The basic equation is (5)

$$D\nabla^2\phi_n\ (th,\underline{r}) \quad - \quad \Sigma_a\ (th,\underline{r}) \ + \ S_n(th,\underline{r}) = \partial_n/\partial t \qquad 38$$

where n is the number of thermal neutrons per cc., S_n is the volume-source of thermal neutrons (due to slowing down of fast neutrons) in neutrons per cc. per second, D is the neutron diffusion coefficient, Σ_a is the neutron absorption coefficinet (the so-called "macroscopic" absorption cross section), and t denotes time. In the steady state, $\partial n/\partial t$ is zero; in the fuel, S_n is zero. The solution, in polar coordinates (corresponding to the case of an infinitely long fuel rod immersed in a uniform external flux), is

$$\phi_n\ (th,r) \ = \ BI_o\left[r\ (\Sigma_a/D)^{\frac{1}{2}} \right] \qquad 39$$

where r is the distance from the axis of the fuel rod and B is an arbitrary constant whose magnitude is proportional to the power level at which the reactor is operating and which may be evaluated by multiplying the gross flux distribution, $f(\underline{r})$ of Equation 33, by the local variation $\phi_n(th,r)$ and proceeding through the operations indicated in Equations 34,35, and 36 to evaluate the product $AB\phi_n(th,Q)$ in terms of Q_f.

It is customary, and conservative practice, to assume that all of the fission energy (200 mev per fission) is released in the fuel, even though the fast neutrons and a significant fraction of the fission gammas and fission-fragment decay gammas release their energy elsewhere. In estimating heat release in moderator, coolants, pressure shells, and shields, however, it is necessary to estimate these leakages from the fuel rather precisely. A fair estimate of the fast neutron spectral distribution may be had from

the Fermi age treatment (5) of simple models; e.g., an infinite, heterogeneous matrix with negligible capture of fast neutrons in U^{235} and U^{238} and only weak capture in the moderator. It is also assumed that the flux of uncollided neutrons from fission is uniform throughout the moderator.

Alternatively, the Monte-Carlo technique (17) permits treatment of the spatial variation of the fast flux for the case of strong absorption in fuel, moderator, and control elements, and leakage out of the reactor, provided the fission rate density in the fuel can be estimated by some means, e.g., by a multi-group diffusion calculation on a "homogenized" model of the core. In the Monte-Carlo method, the events in a neutron "life history" are selected by sampling from a table of random numbers. Initial neutron energies are selected from the fission neutron spectral distribution, a flight direction is established, the coordinates of the first collision are found, kind of nucleus and type of collision is selected, post-collision neutron energy, and new flight direction are determined. The history is followed until the neutron is absorbed or leaks out of the reactor. The process is repeated until a sufficiently large number of cases has been obtained to permit a statistical analysis to obtain the fast flux spatial and spectral distribution.

The technique is applicable to the calculation of fast fluxes in moderator, core vessels, shields, reflectors, thermal shields, pressure shells, biological shields, etc. Unless simplifications are introduced, especially geometric symmetries, an excessive number of histories may be required, even though an electronic computer is used.

GAMMA RAY FLUX DENSITY DISTRIBUTIONS

Compton scattering greatly complicates the estimation of gamma flux densities. The process is not isotropic (Equation 27), and the energy decrement of the scattered photon is strongly dependant on the scattering angle (Equation 2). Photons scattered through small angles retain most of their original energy and may penetrate to remote regions of the reactor. Photons scattered through larger angles suffer large energy losses, and their penetrating power is greatly reduced.

The important gamma sources are the primary gamma sources (the fission process, the decay of fission fragments, neutron captures, inelastic scattering collisions of neutrons) and the secondary source (Compton scattering of photons). At many points of interest, indeed, at most, the contribution to the gamma flux made by scattered photons is many times greater than that of unscattered photons.

In principle, the gamma flux may be constructed by integration over the gamma sources. Let $\underline{r}$ denote the vector coordinates of a point in the reactor, and let $\phi_\gamma (E_\gamma, \underline{r})$ denote the gamma flux density in photon-cm. per second per cc. per mev. Let $\underline{r}'$ denote another point in the reactor, let a volume increment about $\underline{r}'$ be denoted by dV', and let the gamma source strength in photons per second per cc. at $\underline{r}'$ be denoted by $S_\gamma (\underline{r}')$. Let $P(E_\gamma, \underline{r}')$ be the conditional probability, per mev., that a photon will be emitted from, or scattered out of dV' with energy E_γ. Let $P(\underline{r}', \underline{r})$ be the probability, per steradian, that the photon will be emitted in the direction of $\underline{r}$. The probability, per cm. of travel, that a photon will be removed from the beam, denoted by $\mu_t (E_\gamma)$, is simply the sum of the probabilities of the photoelectric, pair production, and Compton scattering processes.

$$\mu_t(E_\gamma) = \mu_{pe}(E_\gamma) + \mu_{pp}(E_\gamma) + \mu_c(E_\gamma) \qquad 40$$

The probability, $P(\underline{R}' \rightarrow \underline{r})$, that a photon will arrive uncollided at $\underline{r}$ is thus simply (in homogeneous, isotropic media)

$$P(\underline{r}' \longrightarrow \underline{r}) = \exp\left[-\mu_t(E_\gamma)\,(|\underline{r}' - \underline{r}|)\right] \qquad 41$$

where $|\underline{r}' - \underline{r}|$ denotes the scalar distance between $\underline{r}'$ and $\underline{r}$. It follows that

$$\phi_\gamma(E_\gamma, \underline{r}) = \int_V S_\gamma(\underline{r}')\,P(E_\gamma,\underline{r}')dV'P(\underline{r}',\underline{r})\exp\left[-\mu_t(E_\gamma)\,(|\underline{r}'-\underline{r}|)\right] \qquad 42$$

The primary gamma sources are isotropic, and for these $P(\underline{r}',\underline{r})$ has the value $1/4\pi$ per steradian. In the case of the secondary gamma source, $P(\underline{r}',\underline{r})$ is a complicated function of the "directed" gamma flux $\phi_\gamma(E_\gamma, \vec{\Omega}, \underline{r}')$, which is defined as the gamma flux at $\underline{r}'$ contributed by photons of energy E moving in the direction $\vec{\Omega}$. It has units of neutron-cm. per second per cc. per mev per steradian. Thus the construction of the flux $\phi_\gamma(E_\gamma, \underline{r})$ at $\underline{r}$ requires a knowledge of the more complicated flux $\phi_\gamma(E_\gamma, \vec{\Omega}, \underline{r}')$ everywhere else. Consequently, a general formulation of $\phi_\gamma(E_\gamma, \underline{r})$ does not seem feasible.

The Monte-Carlo technique may be applied to this problem (14). Suppose the histories of sufficient primary photons of energy E' are determined to define $F(E'; \underline{r}' \rightarrow \underline{r})$, the average fraction of the energy E' of photons originating at $\underline{r}'$ that is dissipated at $\underline{r}$ per cc. The heating rate is now readily formulated as follows.

$$G_\gamma = \int_V \int_{E'_\gamma = 0}^{\infty} S_{p\gamma}(\underline{r}')dV'\,P(E'_\gamma)\,dE'_\gamma\,E'_\gamma\,F(E'_\gamma;\underline{r}' \rightarrow \underline{r}) \qquad 43$$

where $S_{p\gamma}(\underline{r}')$ is the primary gamma source strength.

In principle, $F(E'_\gamma;\underline{r}' \rightarrow \underline{r})$ may be evaluated for any case howsoever complicated in geometry or heterogeneity, but the sheer magnitude of the computational labor may render the method impractical. The simplest case is that of an infinite, homogeneous, isotropic medium. Life histories are generated in the usual manner and an account is kept of the radial distance of each collision from the point of origin and the amount of energy deposited. From these data it is easy to calculate the fraction f(E',R) dR of the energy E' of the original photons that is released inside a spherical annulus of radius R and thickness dR. The volume of such an annulus is $4\pi R^2 dR$, from which it follows that $F(E';\underline{r}' \rightarrow \underline{r})$ is given by

$$F(E';\underline{r}' \rightarrow \underline{r}) = \frac{f[E';(|\underline{r}' - \underline{r}|)]}{4\pi(|\underline{r}' - \underline{r}|)^2} \qquad 44$$

A brilliant, analytical solution of the foregoing case has been obtained by Goldstein and Wilkins (6) along lines laid down by Spenser and Fano (19). Briefly, the method consists in expanding the directed flux, $\phi_\gamma(E_\gamma, \vec{\Omega}, \underline{r})$, in a series the terms of which are products of Laguerre and Legendre polynomials. The directed flux is integrated with respect to all directions to remove the angular dependency, multiplied by the distance from the origin raised to integral powers, and the product is integrated over all space to eliminate the spatial dependency. The integration yields the so-called "moments" of the flux, which are used to evaluate the arbitrary coefficients in the original series in the usual way by application of the boundary

conditions and utilization of the orthogonal properties of the Legendre polynomial. Having constructed the flux $\phi_\gamma(E''_\gamma, \underline{r})$ due to a monoenergetic, unit, point source (one photon per second of energy E') at $\underline{r}'$, Goldstein then computes the fraction of the energy E' released at $\underline{r}$. This, of course, is identical with $F(E'_\gamma, \underline{r}' \rightarrow \underline{r})$ defined above. The results are reported in terms of an "energy absorption build up factor", $B_a(E_o, \mu_o r)$, where E_o is E' in the notation used here, μ_o is $\mu_t(E')$, r is $(|\underline{r}' - \underline{r}|)$, and where B_a is defined in such a way that

$$F(E', \underline{r}' \rightarrow \underline{r}) = \frac{B_a(E', |\underline{r}' - \underline{r}|)}{4\pi(|\underline{r}' - \underline{r}|)^2} \mu_e(E') \exp\left[-\mu_t(E')(|\underline{r}' - \underline{r}|)\right] \qquad 45$$

where μ_e is the energy deposition coefficient defined in Equation 32. Values of B_a as a function of E'_γ and $|\underline{r}' - \underline{r}|$ are tabulated for a variety of materials by Goldstein (6). Taylor (20) has proposed an empirical correlation exponential in $\underline{r}' - \underline{r}$ which is conveniently applied to problems in gamma heating in pressure shells and thermal shields (1).

Conceivably, the function $F(E', |\underline{r}' - \underline{r}|)$ could be evaluated experimentally. Small radiation sources of various energies could be moved about in a mock-up of the system under study and the heat liberation at various points of interest measured. If a direct thermal measurement is made, the measuring instrument would have to be extremely sensitive or else the radiation source strength would have to be inconveniently large. If an electronic measurement is made, the spectral distribution of the gamma flux would have to be measured in order that the heating might be computed by Equation 43.

The mathematical difficulties in estimating gamma heating may be considerably reduced, albeit at an unknown cost in accuracy, if one is

willing to utilize the straight-ahead scattering approximation, investigated carefully by Hurwitz, et al. (11). The assumption is made that photons suffer energy degradations according to Equations 25 and 27, but that the scattered photons fall into one of two classes: (a) photons scattered through such small angles that the deviations from the line of flight may be ignored, and (b) photons scattered through such large angles that the resulting low-energy photons have little penetrating power (high μ_t) and are absorbed near the point where they suffered the scattering. These assumptions depend upon the following arguments: (a) the energy distribution of the scattered photons shows a strong forward component, (b) the angular distribution also shows a strong forward component, (c) distances of interest in heat generation calculations are measured in relatively few attenuation lengths from the primary source, and the probability that a photon will suffer many scatterings is small, (d) photoelectric absorption increases strongly with decreasing photon energy. For example, of the photons scattered out a beam of 6 mev photons, those scattered through angles less than 50 degress carry 90 per cent of the energy of all the scattered photons.

The approximation is applied by breaking the gamma source spectrum into a number of energy groups. The group having the highest energy E_1 is attenuated between $\underline{r}'$ and $\underline{r}$ by the simple exponental factor $\exp\left[-\mu_t(E_1')(|\underline{r}' - \underline{r}|)\right]$. The next group loses photons in proportion to $\mu_t(E_2')$, but gains photons from group 1 by Compton scattering. Group 3 receives photons from both groups 1 and 2, and so on. The transfer of photons between groups is a function of the energy-widths of the

groups and their separation in energy, and is computed from the Klein-Nishina relation (Equation 27). The differential equations expressing the photon "economy" for each group are integrated successively, giving the flux at $\underline{r}$.

Hurwitz, using this multi-group technique, has calculated the energy release ar $\underline{r}$ due to an energy-distributed source at $\underline{r}'$ having a typical spectrum and found that the results could be expressed in terms of an average "effective" gamma absorption coefficient, $\bar{\mu}_{eff}(\bar{E}_\gamma)$, obtained by averaging μ_{eff} over the spectrum of the source

$$\bar{\mu}_{eff}(\bar{E}\gamma) = \int_0^\infty \mu_{eff}(E'_\gamma) P(E'_\gamma)\, dE'_\gamma \qquad 46$$

where $P(E')$ is the spectral distribution and where

$$\mu_{eff}(E'\gamma) = \mu_e(E'_\gamma) + \mu'_c(E'_\gamma) \qquad 47$$

Here μ'_c is a term added to the energy deposition coefficient μ_e to account for the energy release due to gamma photons scattered to energies below 0.25 mev, which are assumed to be absorbed in the near vicinity of their scattering. It is evaluated by integrating the scattered energy ratio E'_γ/E_γ given by Equation 25 with respect to the angular probability given by Equation 27 as follows:

$$\mu'_c(E) = \int_{\theta=0}^{\theta(E'=0.25)} (E'_\gamma/E_\gamma) P(\theta)\, d\theta \qquad 48$$

Hurwitz's results may be put in the form, using the present notation,

$$G_\gamma(\underline{r}) = \int_V \frac{S_{p\gamma}(\underline{r}')\, dV'\, \bar{E}_\gamma \bar{\mu}_{eff}(\bar{E}'_\gamma)}{4\pi(|\underline{r}' - \underline{r}|^2} \exp\left[-\bar{\mu}_{eff}(\bar{E}')(|\underline{r}'-\underline{r}|)\right] \qquad 49$$

PRIMARY GAMMA SOURCES

The fission gamma and fission fragment decay gamma sources are both proportional to the fission density in stationary fuel reactors.

$$S_{p\gamma}(\underline{r}') = N_\gamma \int_0^\infty \phi_n(E_n, \underline{r}') \Sigma_f(E_n, \underline{r}')\, dE_n \qquad 50$$

where N_γ is the number of photons, fission or decay as the case may be ~~by~~, emitted per fission. Gamble (4) reports that 7.5 prompt gamma photons are released per fission, that these have an average energy of about 1 mev, each, and that their spectral distribution is approximately exponential.

$$P(E_\gamma) = \exp (- E_\gamma) \qquad 51$$

This relation does not hold at the extremes of the energy range; however, the contribution from photons with energy above ten mev. is negligible, while gammas emitted with energies less than 0.1 mev, carry less than one per cent of the total energy.

The fission fragment decay gamma spectrum may be constructed from a knowledge of the fission yields of the various nuclides and their decay schemes. Moteff (13) has tabulated these data for some nuclides which have appreciable fission yields, which emit gammas with energies in excess of 0.04 mev., and which have half-lifes in excess of 30 seconds. Several other radioactive fission products are known, and it seems likely that considerable gamma energy is radiated by nuclides having half-lifes lying between Gamble's "prompt" period (0.4 microseconds) and those which have been observed. In all, about 6 mev. is released per fission(21).

The situation is fairly satisfactory with respect to data on the spectrum of the capture gamma rays emitted by a nucleus in returning

to the ground state following absorption of a neutron. Mittleman (12) has tabulated the information available as of October, 1953. The strength of this gamma source is proportional to the local neutron flux and to the capture coefficient Σ_c. Equation 50 may be used by replacing Σ_f by Σ_c.

Hardly any data are available on the spectrum of gamma rays emitted following the fast neutron inelastic collisions. Grace, et al.(7) report some measurements on the scattering of 2.5 mev neutrons. Blizard (3) states that it may be possible to predict these spectra in the same way that spectra of ordinary gamma emitters are predicted (8), but nothing seems to have been published on this.

CONCLUSION

The local rates of heat release in nuclear reactors have been formulated in terms of the local neutron and gamma fluxes. Certain special methods of estimating these fluxes were indicated and their application to the heating problem discussed briefly.

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